EXERCISE ON CONFIDENCE INTERVALS

1. The standard deviation of limestone drilling bits used in oil industry is known to be 24 hours. A random sample of 25 drilling bits was found to have a mean lifetime of 300 hours. Calculate a 95% confidence interval for the population mean.

2. The owner of a large banana plantation wants to know the average yield per tree. He takes a random sample of 100 trees and finds that the sample mean is 130.8 bananas with standard deviation 28.4.

   Assuming that the sample standard deviation is a good estimate for the population standard deviation, construct a 95% confidence interval for the population mean.

3. The characteristics of transistors produced by a single production line are highly variable. The transistors are coded A, B or C according to certain characteristics. From a sample of 80, 60 were found to merit code A. Calculate a 95% confidence interval for the proportion of transistors being produced which are of type A.

4. A market research company responsible for collecting viewing figures for a television company believes that the mean number of hours spent by the viewing public watching the company's output has changed but that the standard deviation has remained constant at 10.5 hours per week. The market research company conducts a survey of 250 randomly selected viewers and finds the sample mean to be 38.6 hours per week. Calculate a 90% confidence interval for the population mean.

5. The daily yield of a chemical manufactured at a certain chemical plant is recorded for 50 consecutive days. The sample mean and standard deviation were found to be 571 tons and 27 tons respectively. Find a 99% confidence interval for the average daily yield, μ tons.

6. From a sample of 150 students sitting a national examination, 30 were awarded a grade A. Calculate a 95% confidence interval for the proportion of all students sitting the examination who would be awarded a grade A.

Exercise on Confidence Intervals

1. X = lifetime of limestone drill bit, in hours

$Var(X) = 24^2$

sample size 25

$\bar{x} = 300$

we know parent population st. deviation, but not its distribution.

as $n > 20$, we use CLT to state $\bar{X} \sim$ approx $N\left(\mu, \frac{24}{25}\right)$

where $\bar{X}$ = mean lifetime from sample of size 25

and μ is (unknown) population mean.

So 95% CI for μ is $\bar{x} \pm z_{0.975}\sqrt{\frac{24^2}{25}}$

$= 300 \pm 1.96\sqrt{\frac{24^2}{25}}$ by invNorm (0.975)

$= 300 \pm 9.40783$

$= (290.592, 309.408)$

$\approx \underline{\underline{(290.6, 309.4)}}$

2. X = yield for one tree

$n = 100$

$\bar{x} = 130.8$

$s_{n-1} = 28.4$

assume $E(X) = \mu$ and $Var(X) = \sigma^2$

due to large sample size, use CLT to state

$$\bar{X} \sim \text{approx } N\left(\mu, \frac{\sigma^2}{100}\right)$$

where $\bar{X}$ = mean yield from sample size 100

so $$\frac{\bar{X} - \mu}{\sqrt{\frac{\sigma^2}{100}}} \approx N(0, 1^2)$$

if we estimate σ with s_{n-1}, we ought to use t distribution

so $$\frac{\bar{X} - \mu}{\sqrt{\frac{s_{n-1}^2}{100}}} \approx t_{99}$$

But t_{99} is very close to Z distribution

$t_{99, 0.975} = 1.98422$ by invt(0.975, 99)

$Z_{0.975} = 1.95996$ by invNorm(0.975)

so 95% CI using Z distribution is $\bar{x} \pm Z_{0.975}\sqrt{\frac{s_{n-1}^2}{100}}$

$= 130.8 \pm 1.95996\sqrt{\frac{28.4^2}{100}}$

$= \underline{(125.234, 136.366)}$

and 95% CI using t distribution is $\bar{x} \pm t_{99, 0.975}\sqrt{\frac{s_{n-1}^2}{100}}$

$= 130.8 \pm 1.98422\sqrt{\frac{28.4^2}{100}}$

$= \underline{(125.165, 136.435)}$

so, to 1dp, by either method, 95% CI is $\underline{\underline{(125.2, 136.4)}}$

3. sample of size 80 had 60 coded A

sample proportion, $\hat{p} = \frac{60}{80}$

let X = number coded A

$X \sim B(80, p)$

Approx with Normal distribution as $np > 5$ and $nq > 5$.

let $Y \sim N(80p, 80pq)$

let $\frac{Y}{80}$ = proportion coded as 'A'

So $\frac{Y}{80} \sim N\left(p, \frac{pq}{80}\right)$

So 95% CI for p is $\hat{p} \pm z_{0.975}\sqrt{\frac{\hat{p}\hat{q}}{80}}$

$= \frac{60}{80} \pm 1.96\sqrt{\frac{\frac{60}{80} \times \frac{20}{80}}{80}}$ by invNorm (0.975)

$= (0.655114, 0.844886)$

$\approx (0.655, 0.845)$

4. X = number of hours of TV watched

$Var(X) = 10{\cdot}5^2$

survey sample size, $n = 250$

Sample mean, $\bar{x} = 38{\cdot}6$

assume $E(X) = \mu$, unknown

as sample so large, by CLT we state $\bar{X} \sim_{approx} N\left(\mu, \frac{10{\cdot}5^2}{250}\right)$

where $\bar{X}$ = mean hours of TV watched

so 90% CI for μ is $\bar{x} \pm Z_{0{\cdot}95}\sqrt{\frac{10{\cdot}5^2}{250}}$

$= 38{\cdot}6 \pm 1{\cdot}645\sqrt{\frac{10{\cdot}5^2}{250}}$ by invNorm $(0{\cdot}95)$

$= (37{\cdot}5077, 39{\cdot}6923)$

$\approx \underline{\underline{(37{\cdot}5, 39{\cdot}7)}}$

5. X = yield of chemical in one day, in tons

sample of size $n = 50$

$\bar{x} = 571$

$s_{n-1} = 27$

let $E(X) = \mu$, $Var(X) = \sigma^2$

let $\bar{X}$ = mean daily yield, from sample of size 50

by CLT, $\bar{X} \sim \text{approx } N\left(\mu, \frac{\sigma^2}{50}\right)$

$$\text{So } \frac{\bar{X} - \mu}{\sqrt{\frac{\sigma^2}{50}}} \sim N(0, 1^2)$$

as we estimate σ with s_{n-1}, we use t-distribution.

$$\text{so } \frac{\bar{X} - \mu}{\sqrt{\frac{s_{n-1}^2}{50}}} \sim t_{49}$$

but, for 99% CI, $z_{0.995} = 2.57583$ by invNorm (0.995)

$t_{49, 0.995} = 2.67995$ by invt (0.995, 49

$\Big\}$ very similar as $t_{49} \approx N(0, 1^2)$

$\therefore$ 99% CI for μ is $\bar{x} \pm z_{0.995}\sqrt{\frac{s_{n-1}^2}{50}}$ or $\bar{x} \pm t_{49, 0.995}\sqrt{\frac{s_{n-1}^2}{50}}$

$= 571 \pm 2.575\sqrt{\frac{27^2}{50}}$ $\quad = 571 \pm 2.67995\sqrt{\frac{27^2}{50}}$

$= (561.165, 580.835)$ $\quad = (560.767, 581.233)$

Therefore, to the nearest ton, we have $\underline{\underline{(561, 581)}}$

6. $n = 150$

grade A awarded to 30 $\Bigg\}$ sample proportion, $\hat{p} = \frac{30}{150}$

let X = number of students gaining grade A

$X \sim B(150, p)$

approximate to Normal distn, as $np > 5$ and $nq > 5$

so $Y \sim N(150p, 150pq)$

let $\frac{Y}{150}$ = proportion of students gaining grade A

$\frac{Y}{150} \sim N\left(p, \frac{pq}{150}\right)$

so 95% CI for p is $\hat{p} \pm z_{0.975}\sqrt{\frac{\hat{p}\hat{q}}{150}}$

$$= \frac{30}{150} \pm 1.95996\sqrt{\frac{\frac{30}{150} \times \frac{120}{150}}{150}} \qquad \text{from invNorm}(0.975)$$

$$= (0.135988, 0.264012)$$

$$\approx \underline{\underline{(0.136, 0.264)}}$$