

The Mann-Whitney Test

In a laboratory, organisms are grown in glass flasks. These flasks can either be left stationary, or aerated by continuously being shaken. To compare the effects of these two growing conditions, the generation times of individual creatures, in hours, are measured:

Aerated	5.5	4.8	3.7	8.1	5.2	
Stationary	8.2	5.6	7.9	5.4	7.1	6.5

We wish to know whether aerating the flasks has a shortening effect on the generation time of organisms.

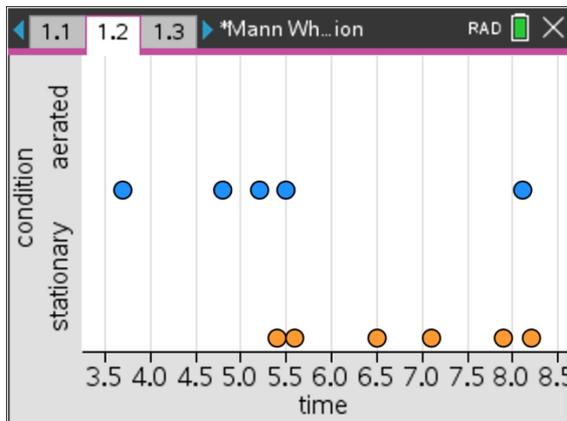


We shall assume that this laboratory's results are a random sample from the population of similarly conducted experiments.

We have non-paired data, so a Mann-Whitney test is potentially appropriate.

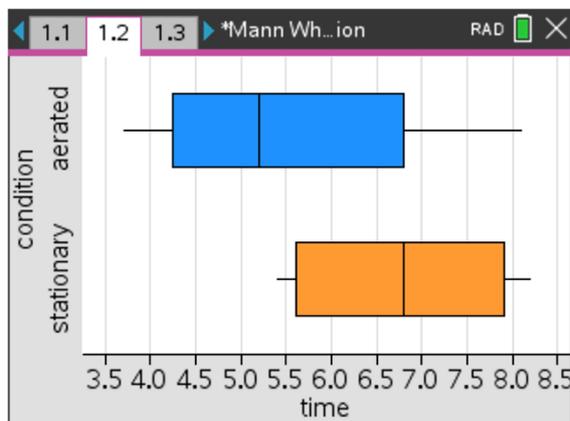
For the Mann-Whitney test we have to **assume** that the population distributions from which the samples have come from have the **same shape and spread**. This allows us to perform a test on the population medians of the two samples.

We can check the plausibility of this assumption by visualisations of the samples:



aerated		stationary
7		3
8		4
5 2		5 4 6
6		5
7		1 9
1		8 2

key: 6|5 = 6.5 hours



The dotplot, boxplot and stem-and-leaf diagram all show that our assumption about the distributions of the two populations, from which these samples have come, is plausible.

We now proceed with the hypothesis test....

the null hypothesis is:

H_0 : population median time for stationary = population median time for aerated

the alternative hypothesis is:

H_1 : population median time for stationary > population median time for aerated

We shall assume H_0 to be true.

We shall decide the significance level of our test, $\alpha = 5\%$

We confirm that this is a 1 tailed-test (because H_1 has a '>' or a '<' in it, and not '≠')

We note that aerated flasks have the smaller sample size, so $m = 5$

Therefore the stationary flasks have the larger sample size, so $n = 6$.

We order all the sample data in ascending order, keeping tracking of which group each piece of data comes from, and then we rank them in order of size:

hours	3.7	4.8	5.2	5.4	5.5	5.6	6.5	7.1	7.9	8.1	8.2
group	A	A	A	S	A	S	S	S	S	A	S
rank	1	2	3	4	5	6	7	8	9	10	11

the rank sum for aerated, $W_a = 1 + 2 + 3 + 5 + 10 = 21$

the rank sum for stationary, $W_s = 4 + 6 + 7 + 8 + 9 + 11 = 45$

We use the rank sum from the **smallest sample size**, which is $m = 5$, the aerated data.

So we proceed and use $W_a = 21$, which is known as a 'test statistic'.

Now, W_a being 21 is just one possible value of the whole range of possible values that the rank sum of a group of 5 objects from a group of 11 objects could take.

We need to find out whether 21 is an unusually low or high value, if the null hypothesis, H_0 , really is true.

We therefore build up the theoretical distribution of all possible values that W_a could take, where there is a group of 5 numbers and a different group of 6 numbers.

If we had different data values in our samples, then the smallest possible value that W_a could take is $1 + 2 + 3 + 4 + 5 = 15$.

The largest possible value that W_a could take is $7 + 8 + 9 + 10 + 11 = 45$

We need to find all of the other possible values that W_a could take, adding up different combinations of 5 numbers, from 11 numbers.

	1	2	3	4	5	6	7	8	9	10	11
15	✓	✓	✓	✓	✓						
16	✓	✓	✓	✓		✓					
17	✓	✓	✓	✓			✓				
17	✓	✓	✓		✓	✓					
18	✓	✓	✓	✓				✓			
18	✓	✓	✓		✓		✓				
18	✓	✓		✓	✓	✓					

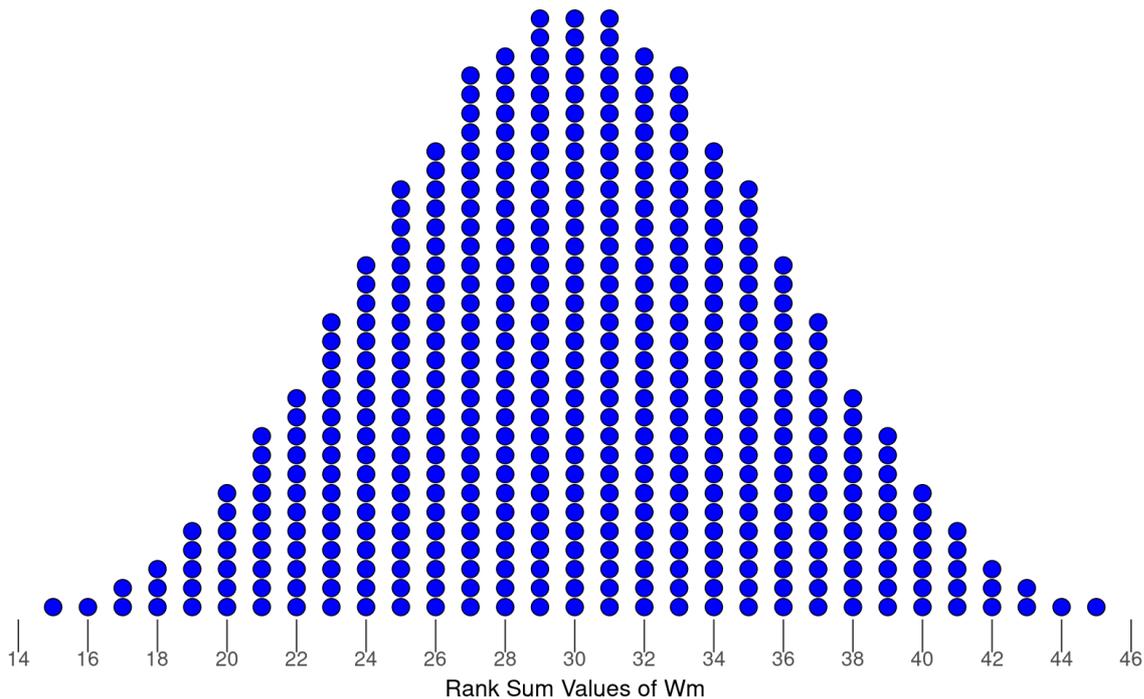
etc etc

Visit <https://nhost.shinyapps.io/MannWhitneyWilcoxonDistributions/>

Select 'Mann-Whitney Distributions'

Select $m = 5$ and $n = 6$

Select 'lower tail'



Variation 1 - using critical values

Return to <https://nhost.shinyapps.io/MannWhitneyWilcoxonDistributions/>

Select 'lower tail'

Select a significance level of '5%'

It displays '5% critical value for lower tail = 20'

This means that the most extreme lower 5% of possible values are those whose rank sums are 20, or less.

We measured $W_a = 21$, which is not in this 5% critical region and we have no evidence to reject H_0 .

This is a consistent result with the first conclusion.

Variation 2 - changing significance level

Return to <https://nhost.shinyapps.io/MannWhitneyWilcoxonDistributions/>

Select a significance level of '10%'

It now displays '10% critical value for lower tail = 22', and the region that is shaded red has now become larger.

This means that the most extreme lower 10% of possible values are those whose rank sums are 22, or less.

We measured $W_a = 21$, which is in the 10% critical region and we now do have evidence to reject H_0 .

This means that if we had chosen a 10% significance level at the very beginning of this analysis, our conclusion would now read as either of the following:

$P(W_a \leq 21) = \frac{29}{462} \approx 0.06 < 0.10$ and so our rank sum value of $W_a = 21$ is in the most extreme 10% of possible values that it could take.

We therefore reject H_0 , in favour of H_1 .

We conclude that we have evidence to suggest that the population median times for stationary flasks are greater than the population median times for aerated flasks, and so there does appear to be a shortening effect on the generation times when aerating.

or

As $W_a = 21 \leq 22$, the 10% critical value, we reject H_0 , in favour of H_1 .

We conclude that we have evidence to suggest that the population median times for stationary flasks are greater than the population median times for aerated flasks, and so there does appear to be a shortening effect on the generation times when aerating.

Variation 3 - exploring other situations

Return to <https://nhost.shinyapps.io/MannWhitneyWilcoxonDistributions/>

Change the controllers for m , n , the **Alternative Hypothesis** and the **Significance Level** to see the effects that each of these can have on the critical values.

Your teacher will show you how these are recorded in the AH Statistical Formulae and Tables Booklet.