Wilcoxon Signed Rank Test on Single Samples

The Wilcoxon Signed Rank Test in the AH Statistics Course requires the assumption that the population distribution of the differences is symmetrical. This then supports the hypothesis test's focus upon the median, rather than any other measure of location.

The Wilcoxon Signed Rank Test analyses the **median of differences** from paired data.
This is **not** the same as testing the **difference in the medians**.
In general, a median difference of zero is **not** the same as the medians being equal to each other. Consider the following example data sets to demonstrate this:

Example 1

Data set, A	2	4	5	6	8
Data set, B	3	8	3	5	6
A-B	-1	-4	2	1	2

So, median(A) = median(B), but median(A-B) $\neq$ 0

Example 2

Data set, C	2	4	5	6	8
Data set, D	3	8	4	6	8
C-D	-1	-4	1	0	0

So, median(C) $\neq$ median(D), but median(C-D) = 0,

If the '**difference in the medians'** is used, then one is ignoring the pairing of the data, whilst '**median of differences'** uses the pairings.

The single sample Wilcoxon Signed Rank Test has a similar approach to the paired sample Wilcoxon Signed Rank Test, in that it also analyses the **median of differences**.

For the single sample Wilcoxon Signed Rank Test, a second data set of a constant value is created in order to match up with each member of the first data set.
Therefore, whilst a set of 'paired data' is created, ignoring the pairing does not actually impact upon the calculations and subsequent statements about medians.

Example 3

Data set, E	2	4	5	6	8
Data set, F	5	5	5	5	5
E-F	-3	-1	0	1	3

So, median(E) = median(F) and also median(E-F) = 0
Or, put another way, 'median(difference of E from constant) = 0'
is equivalent to 'median(E) = constant'

Example 4

Data set, G	2	4	5	6	8
Data set, H	3	3	3	3	3
G-H	-1	1	2	3	5

So, median(G) = 5 and median(H)=3 and median(G-H) = 2.
Or, put another way, 'median(difference of G from constant) = k'
is equivalent to 'median(G) = constant + k'

Therefore, for **single sample** Wilcoxon Signed Rank Tests **only**, we can write a hypothesis statement of the form 'median difference from constant = 0'
in the simpler form of 'median = a constant'

Four questions with full worked solutions.

Question 1

A special questionnaire is designed to assess the amount of psychological trauma suffered by individuals following a tragedy. The median score on this questionnaire for those who have not been recently involved in a disaster is 10, while higher scores are reporting by those who have suffered trauma. A random sample of emergency services personnel who had attended a recent motorway accident completed the questionnaire and their scores were:

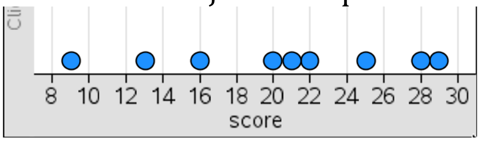
20	29	13	16	9	22	28	21	25

Investigate whether the median score on this questionnaire is greater than 10 for emergency services personnel who have recently attended motorway accidents.

Solution overleaf....

Solution 1

First, we plot the data to obtain an initial subjective impression:



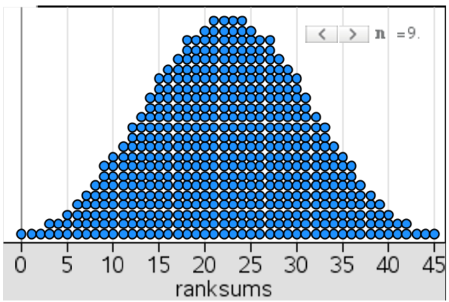
It would appear as though the median score is greater than 10

Validity: we have single sample data and we are concerned with the median
Assumptions: we assume that the parent distribution of questionnaire scores is symmetrical in shape

H_0: $\text{median}_{\text{SCORE}}=10$
H_1: $\text{median}_{\text{SCORE}}>10$
Alpha = 5%   One tail test.
We assume H_0 to be true

score	20	29	13	16	9	22	28	21	25
Ho	10	10	10	10	10	10	10	10	10
score-Ho	10	19	3	6	-1	12	18	11	15
|score-Ho|	10	19	3	6	1	12	18	11	15
rank	4	9	2	3	1	6	8	5	7

W_{pos} = 2+3+4+5+6+7+8+9 = 44
W_{neg} = 1



We are interested in $P(W_{neg} \leq 1) = \frac{2}{2^9} = 0.00390625 \approx 0.4\%$

Alternatively, from tables for n=9, $P(W_{neg} \leq 8) = 0.05$
$P(W_{neg} \leq 5) = 0.025$
$P(W_{neg} \leq 3) = 0.01$
$P(W_{neg} \leq 1) = 0.005$

So, we are inside the 5% critical region. Our p-value is 0.4% « 5%

So the evidence suggests that the null hypothesis should be rejected and we conclude that the median scores of emergency services personnel who have recently attended motorway accidents is greater than 10.

Question 2

A manufacturer claims that its brand of light bulb has a median life of 1000 hours.
A random sample of 10 of these light bulbs had the following life times (hours)

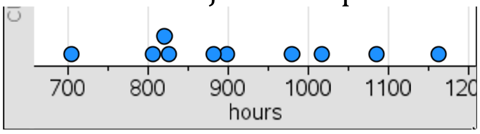
897	1016	980	825	806	1163	881	703	820	1085

Investigate the manufacturer's claim.

Solution overleaf….

Solution 2

First, we plot the data to obtain an initial subjective impression:



It would appear as though the median bulb lifetime is less than 1000

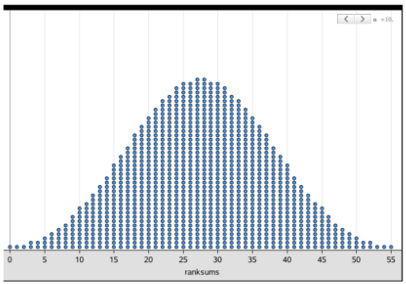
Validity: we have single sample data and we are concerned with the median
Assumptions: we assume that the parent distribution of lifetime hours is symmetrical in shape

H_0: $median_{HOURS}=1000$
H_1: $median_{HOURS}\neq 1000$
Alpha = 5% Two tailed test.
We assume H_0 to be true

hours	897	1016	980	825	806	1163	881	703	820	1085
Ho	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
hours-Ho	-103	16	-20	-175	-194	163	-119	-297	-180	85
|hours-Ho|	103	16	20	175	194	163	119	297	180	85
rank	4	1	2	7	9	6	5	10	8	3

W_{pos} = 1+6+3 = 10
W_{neg} = 4+2+7+9+5+10+8=45

We are interested in $P(W_{pos}\leq 10)$



From tables for n=10,
$P(W_{pos} \leq 10) = 0.05$
$P(W_{pos} \leq 8) = 0.025$
$P(W_{pos} \leq 5) = 0.01$
$P(W_{pos} \leq 3) = 0.005$

Now we are conducting a 2-tailed 5% test, so we **either** compare the above probabilities to 2.5% **or** we obtain the p-value, and compare it to 5%.

Now, in this question we can't easily work out the exact value of $P(W_{pos}\leq 10)$ as we don't know how many points in the shown distribution are less than 11 (we *could* count all the dots, but that will take some time). So we cannot work out the p-value, and thus compare it to the alpha value.

So, we shall have to use the table's $P(W_{pos} \leq 10) = 0.05$
In other words, the rank sum of 10 is on the cusp of the 5% tail, but this means that it's not in the 2.5% tail.
Hence, 10 is not in the two-tailed 5% critical region.
So we have no evidence to reject the null hypothesis and therefore the median lightbulb life is 1000 hours.

Question 3

Some years ago, the weekly incomes, to the nearest pound, of a random sample of people living in a certain housing estate were recorded:

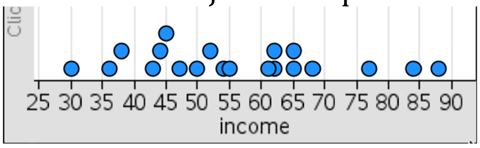
43	88	36	65	62	62	77	50	38	47
61	44	52	84	30	45	65	54	68	55

At the time this sample survey was conducted, the median weekly income in the region containing this housing estate was £68, and it was thought that incomes on the estate were on average less than in the region as a whole.
Investigate this hypothesis.

Solution overleaf....

Solution 3

First, we plot the data to obtain an initial subjective impression:



It would appear as though the median income is less than 68

Validity: we have single sample data and we are concerned with the median
Assumptions: we assume that the parent distribution of income is symmetrical in shape

H_0: $\text{median}_{\text{INCOME}}=68$
H_1: $\text{median}_{\text{INCOME}}<68$
Alpha = 5%   One tail test.
We assume H_0 to be true

income	43	88	36	65	62	62	77	50	38	47	61	44	52	84	30	45	65	54	68	55
Ho	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68	68
income-Ho	-25	20	-32	-3	-6	-6	9	-18	-30	-21	-7	-24	-16	16	-38	-23	-3	-14	0	-13
|income-Ho|	25	20	32	3	6	6	9	18	30	21	7	24	16	16	38	23	3	14	0	13
rank	16	12	18	1.5	3.5	3.5	6	11	17	13	5	15	9.5	9.5	19	14	1.5	8		7

W_{pos} = 12+6+9.5 = 27.5
W_{neg} = 16+18+1.5+3.5+3.5+11+17+13+5+15+9.5+19+14+1.5+8+7=162.5

We are interested in $P(W_{pos} \leq 27.5)$

From tables for n=19, $P(W_{pos} \leq 53) = 0.05$
$P(W_{pos} \leq 46) = 0.025$
$P(W_{pos} \leq 37) = 0.01$
$P(W_{pos} \leq 32) = 0.005$

So, we are well inside the 5% critical region, as $P(W_{pos} \leq 27.5) < P(W_{pos} \leq 32) = 0.005$

So the evidence suggests that the null hypothesis should be rejected and we conclude from this sample that the median income is less than £68

Question 4

It is believed that, on average, 11-year-old boys will take 4 minutes to complete a sorting exercise when wearing a blindfold. The time taken in seconds by a random sample of nine 11-year-old boys to complete the sorting exercise was

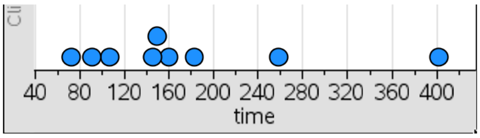
72	258	160	145	107	182	149	91	402

Does this data provide evidence which supports the hypothesis?

Solution overleaf....

Solution 4

First, we plot the data to obtain an initial subjective impression:



It would appear as though the median sorting lifetime is less than 240 seconds

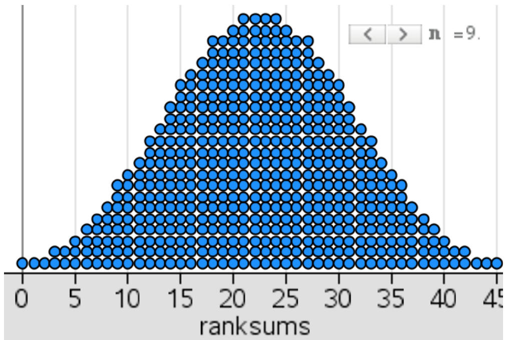
Validity: we have single sample data and we are concerned with the median
Assumptions: we assume that the parent distribution of sorting times is symmetrical in shape

H_0: $\text{median}_{\text{TIME}}=240$
H_1: $\text{median}_{\text{TIME}}\neq240$
Alpha = 5%   Two tailed test.
We assume H_0 to be true

time	72	258	160	145	107	182	149	91	402
Ho	240	240	240	240	240	240	240	240	240
time-Ho	-168	18	-80	-95	-133	-58	-91	-149	162
|time-Ho|	168	18	80	95	133	58	91	149	162
rank	9	1	3	5	6	2	4	7	8

$W_{pos} = 1+8 = 9$
$W_{neg} = 9+3+5+6+2+4+7 = 36$

We are interested in $P(W_{pos}\leq9)$



From tables for n=9, $P(W_{pos} \leq 8) = 0.05$
$P(W_{pos} \leq 5) = 0.025$
$P(W_{pos} \leq 3) = 0.01$
$P(W_{pos} \leq 1) = 0.005$

Now we are conducting a 2-tailed 5% test, so we **either** compare the above probabilities to 2.5% **or** we obtain the p-value, and compare it to 5%.

Now, in this question we can't easily work out the exact value of $P(W_{pos}\leq9)$ as we don't know how many points in the shown distribution are less than 10 (we *could* count all the dots, but that will take some time). So we cannot easily work out the p-value, and thus compare it to the alpha value.

So, we shall have to use the table's $P(W_{pos} \leq 8) = 0.05$
In other words, the rank sum of 9 is outside the 5% tail, and this therefore means that it's certainly outside the 2.5% tail as well.
Hence, 9 is not in the two-tailed 5% critical region.
So we have no evidence to reject the null hypothesis and therefore the median sorting time appears to be 240 seconds, or 4 minutes.