Exercise 7 - Combinations - Worked Solutions

1a) $^7C_3 = \frac{7!}{3!4!}$

$= \frac{7 \times \cancel{6} \times 5 \times \cancel{4 \times 3 \times 2 \times 1}}{\cancel{3 \times 2} \times 1 \times \cancel{4 \times 3 \times 2 \times 1}}$

$= 35$.

b) $^6C_4 = \frac{6!}{4!2!}$

$= \frac{6 \times 5 \times \cancel{4 \times 3 \times 2 \times 1}}{\cancel{4 \times 3 \times 2 \times 1} \times 2 \times 1}$

$= \frac{6 \times 5}{2}$

$= 15$.

c) $^8C_1 = \frac{8!}{7!1!}$

$= 8$.

d) $^5C_0 = \frac{5!}{0!5!}$

$= 1$

2. $\binom{n}{n-r} = \binom{n}{r}$

a) $n=9$
$r=3$

$$\binom{9}{6} = \binom{9}{3}$$

$$LHS = \frac{9!}{6!3!}$$

$$RHS = \frac{9!}{3!6!}$$

$$= LHS.$$

b) $n=7$
$r=4$

$$\binom{7}{3} = \binom{7}{4}$$

$$LHS = \frac{7!}{3!4!}$$

$$RHS = \frac{7!}{4!3!}$$

$$= LHS.$$

3. choosing 3 from 8

ways choosing 3 from 8 $= {}^8C_3$

$$= \frac{8!}{3!5!}$$

$$= \frac{8 \times 7 \times 6}{3 \times 2 \times 1}$$

$$= \underline{\underline{56}}.$$

4. # ways choosing 5 letters from 8 $= {}^8C_5$

$= \dfrac{8!}{5!3!}$

$= \dfrac{8 \times 7 \times 6}{3 \times 2 \times 1}$

$= \underline{\underline{56}}$

5 a) choose 3 from 10 $= {}^{10}C_3$

$$= \frac{10!}{7!3!}$$

$$= \frac{10\times9\times8}{3\times2\times1}$$

$$= 10\times3\times4$$

$$= \underline{\underline{120.}}$$

b) choose 14 from 17 $= {}^{17}C_{14}$

$$= \frac{17!}{14!3!}$$

$$= \frac{17\times16\times15}{3\times2\times1}$$

$$= 17\times8\times5$$

$$= 17\times40$$

$$= \underline{\underline{680}}$$

c) choose 6 from 30 $= {}^{30}C_6$

$$= \frac{30!}{24!6!}$$

$$= \frac{\cancel{30}\times29\times\overset{7}{\cancel{28}}\times\overset{9}{\cancel{27}}\times\overset{13}{\cancel{26}}\times25}{\cancel{6\times5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1}$$

$$= 29\times7\times9\times13\times25$$

$$= \underline{\underline{593775}}$$

d) choose 5 from 52 $= {}^{52}C_5$

$$= \frac{52!}{5!47!}$$

$$= \frac{52\times51\times\overset{10}{\cancel{50}}\times49\times\cancel{48}^{2}}{\cancel{5}\times\cancel{4\times3\times2}\times1}$$

$$= 52\times51\times10\times49\times2$$

$$= \underline{\underline{2\,598\,960}}$$

6. choose 2 from 8 $= {}^8C_2$

$$= \frac{8!}{6!\,2!}$$

$$= \frac{8 \times 7}{2 \times 1}$$

$$= \underline{\underline{28 \text{ ways}}}$$

if no vowels contained, then only choosing 2 from 5 letters (N, T, G, R, L)

$$= {}^5C_2$$

$$= \frac{5!}{3!\,2!}$$

$$= \frac{5 \times 4}{2 \times 1}$$

$$= \underline{\underline{10}}$$

7. choose 7 cards from 13 $= {}^{13}C_7$

$= 1716.$

if one card is ace, then only chosing 6 cards from 12

$= {}^{12}C_6$

$= 924.$

8.

a) choose 5 from 30 $= {}^{30}C_5$

$= \underline{\underline{142506}}.$

b) if oldest chosen by default, then only choosing 4 from 29 $= {}^{29}C_4$

$= \underline{\underline{23751}}$

9.

if any 4 to be chosen from 12, it would be ${}^{12}C_4$

a) if best debater included, then choosing 3 from 11 is ${}^{11}C_3$

$$= \frac{11 \times 10 \times 9}{3 \times 2 \times 1}$$

$$= 11 \times 5 \times 3$$

$$= \underline{\underline{165.}}$$

b) assuming best debater and oldest student are different people then we are only choosing 2 from 10 $= {}^{10}C_2$

$$= \frac{10 \times 9}{2 \times 1}$$

$$= \underline{\underline{45}}$$

10. 8 different biscuits.

buy 4 packets

a) choose 4 packets from 8 $= {}^8C_4$

$$= \frac{\cancel{8} \times 7 \times \overset{2}{\cancel{6}} \times 5}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 7 \times 2 \times 5$$

$$= \underline{\underline{70}}.$$

b) if two of the chosen 4 packets are the same

then there are 8 ways of picking first packet

then 1 way of picking second packet (the duplicate)

then we need to pick 2 from the remaining 7 packets

so # ways $= 8 \times 1 \times {}^7C_2$

$$= 8 \times \frac{7 \times 6}{2 \times 1}$$

$$= 8 \times 21$$

$$= \underline{\underline{168}}$$

11. 9 varieties

pick 4 chocolates if...

a) all 4 are different = choose 4 from 9

$= {}^9C_4$

$= \frac{{}^{3}\cancel{9} \times \cancel{8} \times 7 \times 6 \times \cancel{5}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$

$= 3 \times 7 \times 6$

$= \underline{\underline{126}}$

b) two same and others different = any of the 9 × duplicate × picking 2 from 8

$= 9 \times 1 \times {}^8C_2$

$= 9 \times \frac{8 \times 7}{2 \times 1}$

$= 9 \times 4 \times 7$

$= \underline{\underline{252}}$

c) three the same and fourth different $= 9 \times 1 \times 1 \times {}^8C_1$

$= 9 \times 8$

$= \underline{\underline{72.}}$

12. 12 men

8 women

Committee of 8 chosen

a) # ways for 5 men and 3 women $= {}^{12}C_5 \times {}^{8}C_3$

$= 792 \times 56$

$= \underline{\underline{44352}}$

b) # ways for at least one man and one woman

consider # ways to get all male committee $= {}^{12}C_8$

$= 495.$

ways to get all female committee $= {}^{8}C_8$

$= 1$

ways to form any committee of 8 (which includes single sex committees)

$= {}^{20}C_8$

$= 125970$

so of the 125970 committees, the ones that have mixed gender membership

$= 125970 - 495 - 1$

$= \underline{\underline{125474}}.$