Exercise 8 – Combinations (Probability) – Solutions.

1. P(all 3 girls selected) = $\dfrac{\text{no. ways to pick 3 girls \& 2 boys}}{\text{no. ways chose 5 from 8}}$

$$= \frac{{}^3C_3 \times {}^5C_2}{{}^8C_5}$$

$$= \frac{1 \times \frac{5 \times 4}{2 \times 1}}{\frac{8 \times 7 \times 6}{3 \times 2 \times 1}}$$

$$= \frac{10}{56}$$

$$= \frac{5}{28}.$$

2. Bag has 4 B and 1 W disc.

P(white disc not removed) = P(only black discs removed)

$$= \frac{\text{choose 2 black discs from 4}}{\text{choose 2 discs from 5}}$$

$$= \frac{{}^4C_2}{{}^5C_2}$$

$$= \frac{\frac{4\times3}{2\times1}}{\frac{5\times4}{2\times1}}$$

$$= \frac{3}{5}.$$

3. 4 questions chosen from 8

P(two even numbered questions) = $\dfrac{\text{\# ways to pick two even q}^{ns}\text{ and two odd q}^{ns}}{\text{\# ways to pick four questions}}$

$$= \frac{{}^4C_2 \times {}^4C_2}{{}^8C_4}$$

$$= \frac{\frac{4\times3}{2\times1} \times \frac{4\times3}{2\times1}}{\frac{\cancel{8}\times7\times\cancel{6}^{2}\times5}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1}}$$

$$= \frac{6 \times 6}{7\times2\times5}$$

$$= \frac{36}{70}$$

$$= \frac{18}{35}$$

4. 10 cubes.

4 green

6 yellow

pick 4 cubes

$$P(2 \text{ green and } 2 \text{ yellow}) = \frac{\# \text{ ways pick 2 green \& 2 yellow}}{\# \text{ ways to pick 4 from 10.}}$$

$$= \frac{{}^4C_2 \times {}^6C_2}{{}^{10}C_4}$$

$$= \frac{\frac{4\times3}{2\times1} \times \frac{6\times5}{2\times1}}{\frac{10\times\cancel{9}^{3}\times\cancel{8}\times7}{\cancel{4}\times\cancel{3}\times\cancel{2}\times1}}$$

$$= \frac{{}^{3}\cancel{6} \times \cancel{15}^{5}}{\cancel{10}_{2}\times\cancel{3}\times7}$$

$$= \frac{3}{7}.$$

5. 4 letters from COMPLEX

$$P(\text{both vowels picked}) = \frac{\text{\# ways pick 2 vowels from 2 vowels \& 2 const. from 5}}{\text{\# ways to pick 4 from 7}}$$

$$= \frac{{}^2C_2 \times {}^5C_2}{{}^7C_4}$$

$$= \frac{1 \times \frac{5 \times 4}{2 \times 1}}{\frac{7 \times 6 \times 5}{3 \times 2 \times 1}}$$

$$= \frac{1 \times 10}{35}$$

$$= \frac{10}{35}$$

$$= \frac{2}{7}.$$

6. 5 children from $\underbrace{8 \text{ girls} \& 12 \text{ boys}}_{20 \text{ children.}}$

a) $P(\text{all boys}) = \dfrac{\#\text{ways pick 5 from 12}}{\#\text{ways pick 5 from 20}}$

$$= \frac{{}^{12}C_5}{{}^{20}C_5}$$

$$= \frac{792}{15504}$$

$$= \frac{33}{646}.$$

b) $P(\text{at least one girl}) = 1 - P(\text{no girls})$

$$= 1 - P(\text{all boys})$$

$$= 1 - \frac{33}{646}$$

$$= \frac{613}{646}.$$

7. 8 red.
5 yellow $\Big\}$ 16 sweets
3 green.
pick 2

$$P(\text{both red}) = \frac{\#\text{ ways pick 2 from 8}}{\#\text{ ways pick 2 from 16}}$$

$$= \frac{{}^{8}C_2}{{}^{16}C_2}$$

$$= \frac{\frac{8\times7}{2\times1}}{\frac{16\times15}{2\times1}}$$

$$= \frac{8\times7}{16\times15}$$

$$= \frac{7}{2\times15}$$

$$= \frac{7}{30}.$$

8. 52 cards

4 picked.

a) $P(\text{all black cards}) = \dfrac{\# \text{ ways pick 4 from 26}}{\# \text{ ways pick 4 from 52}}$

$$= \frac{{}^{26}C_4}{{}^{52}C_4}$$

$$= \frac{46}{833}.$$

b) $P(\text{all kings}) = \dfrac{{}^{4}C_4}{{}^{52}C_4}$ ← pick 4 kings from 4

$$= \frac{1}{270725}.$$

c) $P(\text{at least one king}) = 1 - P(\text{no kings})$

$$= 1 - \frac{{}^{48}C_4}{{}^{52}C_4}$$

$$= 1 - \frac{38916}{54145}$$

$$= \frac{15229}{54145}$$

9. 5 green
4 yellow
3 blue

$\left.\begin{matrix} 5 \text{ green} \\ 4 \text{ yellow} \\ 3 \text{ blue} \end{matrix}\right\} 12 \text{ discs.}$

pick 4 discs.

a) $P(\text{exactly 3 blue discs}) = \dfrac{\overbrace{{}^{3}C_3}^{\text{3 blue}} \times \overbrace{{}^{9}C_1}^{\text{1 non-blue}}}{\underbrace{{}^{12}C_4}_{\text{any 4}}}$

$= \dfrac{1 \times 9}{495}$

$= \dfrac{1}{55}$

b) $P(\text{exactly 3 yellow discs}) = \dfrac{\overbrace{{}^{4}C_3}^{\text{3 yellow}} \times \overbrace{{}^{8}C_1}^{\text{1 non-yellow}}}{\underbrace{{}^{12}C_4}_{\text{any 4}}}$

$= \dfrac{4 \times 8}{495}$

$= \dfrac{32}{495}$

c) $P(\text{at least one green}) = 1 - P(\text{no green})$

$= 1 - \dfrac{{}^{7}C_4}{{}^{12}C_4}$ ← 4 non-green.

$= 1 - \dfrac{7}{99}$

$= \dfrac{92}{99}$

10. 52 cards

7 dealt.

a) $$P(4\text{ aces}) = \frac{\underbrace{{}^4C_4}_{4\text{ aces}} \times \underbrace{{}^{48}C_3}_{3\text{ non-aces}}}{\underbrace{{}^{52}C_7}_{\text{any } 7}}$$

$$= \frac{1 \times 17296}{133784560}$$

$$= \frac{1}{7735}.$$

b) $$P(\text{exactly } 3\text{ aces}) = \frac{\underbrace{{}^4C_3}_{3\text{ aces}} \times \underbrace{{}^{48}C_4}_{4\text{ non-aces}}}{\underbrace{{}^{52}C_7}_{\text{any } 7}}$$

$$= \frac{4 \times 194580}{133784560}$$

$$= \frac{9}{1547}$$

c) $$P(\text{at least } 3\text{ aces}) = P(3\text{ aces}) + P(4\text{ aces})$$

$$= \frac{9}{1547} + \frac{1}{7735}$$

$$= \frac{46}{7735}.$$

11. 52 cards
6 dealt

a) $P(\text{all black}) = \dfrac{{}^{26}C_6}{{}^{52}C_6}$ ← 6 black, ← any 6

$= \dfrac{253}{22372}$

b) $P(\text{exactly 5 black cards}) = \dfrac{{}^{26}C_5 \times {}^{26}C_1}{{}^{52}C_6}$ ← 5 black, ← 1 red, ← any 6

$= \dfrac{3289}{39151}$

c) $P(\text{at least 5 black cards}) = P(5\text{ black}) + P(6\text{ black})$

$= \dfrac{3289}{39151} + \dfrac{253}{22372}$

$= \dfrac{14927}{156604}$

12. 8 cards : 1, 2, 3, 4, 5, 6, 7, 8
3 dealt

a) $P(\text{all 3 cards are even}) = \dfrac{{}^4C_3 \leftarrow \text{3 even}}{{}^8C_3 \leftarrow \text{any 3}}$

$= \dfrac{4}{56}.$

$= \dfrac{1}{14}.$

b) $P(\text{product is odd}) = P(\text{all 3 are odd})$

$= \dfrac{{}^4C_3 \leftarrow \text{3 odd}}{{}^8C_3 \leftarrow \text{any 3}}$

$= \dfrac{1}{14}$